## PRE-CALCULUS: by Finney, Demana, Watts and Kennedy Chapter 3: Exponential, Logistic, and Logarithmic Functions

## 3.1: Exponential and Logistic Functions

For those that are exponential functions, state the initial value and the base. For those that are not, explain.

Which of the following are exponential functions?

A)  $f(x) = 3^x$ 

B) 
$$g(x) = 6x^{-4}$$

**Exponential Function** A function that can be rewritten in the form  $y = a \cdot b^x$ , where a is non-zero, b is positive, and  $b \neq 1$ . a: initial value at x=0

b: base

C)  $h(x) = -2 \cdot 1.5^x$ 

D)  $h(x) = 7 \cdot -2^x$ 

E)  $f(x) = 5 \cdot 6^{\pi}$ 

Compute the exact value of the function without using a calculator

A) 
$$2 \cdot 4^x$$
 when  $x = 0$ 

B) 
$$2 \cdot 4^x$$
 when  $x = -3$ 

C) 
$$-2 \cdot 4^x$$
 when  $x = 1/2$  D)  $3 \cdot 8^x$  when  $x = -2/3$ 

D) 
$$3.8^{x}$$
 when  $x = -2/3$ 

2 | Page

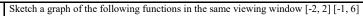
1 | Page

Determine a formula for the exponential function g(x) and h(x) whose values are given in the table

| X  | g(x) | X  | h(x) |
|----|------|----|------|
| -2 | 4/9  | -2 | 128  |
| -1 | 4/3  | -1 | 32   |
| 0  | 4    | 0  | 8    |
| 1  | 12   | 1  | 2    |
| 2  | 36   | 2  | 1/2  |

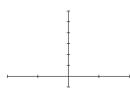
Given 2 points on the graph of an exponential function, find the formula

$$B) (0.3) \left(3, \frac{3}{e}\right)$$



$$y = 2^x$$
  $y = 3^x$   $y = 4^x$   $y = 5$ 

$$y = 2^{x}$$
  $y = 3^{x}$   $y = 4^{x}$   $y = 5^{x}$   $y = \left(\frac{1}{2}\right)^{x}$   $y = \left(\frac{1}{3}\right)^{x}$   $y = \left(\frac{1}{4}\right)^{x}$   $y = \left(\frac{1}{5}\right)^{x}$ 





- 1) Determine the domain and range
- 1) Determine the domain and range
- 2) Is the function even, odd or neither 2) Is the function even, odd or neither
- 3) Intervals of Increase or Decrease
- 3) Intervals of Increase or Decrease

4) Find any extrema.

- 4) Find any extrema.
- 5) Determine the end behavior
- 5) Determine the end behavior

- 6) Find any asymptotes
- 6) Find any asymptotes

- 7) Intervals of Concavity
- 7) Intervals of Concavity

Describe how to transform the graph of  $f(x) = 2^x$  into the graph of g

a) 
$$g(x) = 2^{x-1}$$

b) 
$$g(x) = 2^{-x}$$

$$g(x) = 3 \cdot 2^x$$

d) 
$$g(x) = 2^{3-x}$$

Describe how to transform the graph of  $f(x) = e^x$  into the graph of g

$$a) g(x) = e^{4x}$$

b) 
$$g(x) = e^{-4x}$$

$$g(x) = 3 \cdot e^x + 1$$

d) 
$$g(x) = e^{2-2x}$$

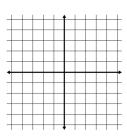
State whether the function is exponential growth or decay and describe its end behavior

A) 
$$f(x) = 2^{3x}$$
 B)  $f(x) = 2^{-3x}$  C)  $f(x) = \left(\frac{1}{4}\right)^{x}$  D)  $f(x) = \left(\frac{1}{4}\right)^{-x}$ 

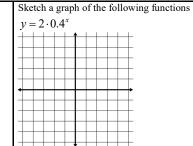
Graph the following functions on your calculator. Find the y-intercept and the horizontal asymptotes

$$f(x) = \frac{8}{1 + 3 \cdot 0.7^x}$$

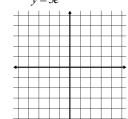
$$f(x) = \frac{20}{1 + 2 \cdot e^{-3x}}$$



**5** | Page



 $y=3e^{-x}$ 



1) Determine the domain and range

1) Determine the domain and range

2) Is the function even, odd or undefined 2) Is the function even, odd or undefined for x < 0

3) Intervals of Increase or Decrease

3) Intervals of Increase or Decrease

4) Find any extrema.

4) Find any extrema.

5) Determine the end behavior

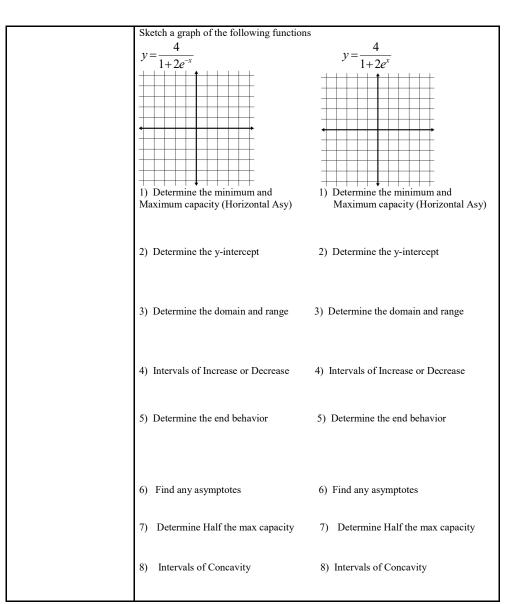
5) Determine the end behavior

6) Find any asymptotes

6) Find any asymptotes

7) Intervals of Concavity

7) Intervals of Concavity



7 | Page

| City 2000 Population 2007 Population Austin, TX 465,622 656,562  San Jose, CA 898,759 939,899  a) When will the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 1 million personal to the population of San Jose, California, surpass 2 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, California, surpass 3 million personal to the population of San Jose, california, surpass 3 million personal to the population of San Jose, california, surpass 3 million personal to the population of San Jose, california, surpass 3 million personal to the population of San Jose, california, surpass 3 million personal to the population of San Jose, california, surpass  | Austin, TX   465,622   656,562   San Jose, CA   898,759   939,899    a) When will the population of San Jose, California, surpass 1 million per |
|--|---|
| a) When will the population of San Jose, California, surpass 1 million personal surpass 1 million personal surpass 1 million personal surpass 2 million personal surpass 3 million surpass 3 mil | a) When will the population of San Jose, California, surpass 1 million per  |
| a) When will the population of San Jose, California, surpass 1 million personal content of the c | a) When will the population of San Jose, California, surpass 1 million per  |
|  |   |
| b) In what year will the population of Austin, TX and San Jose, CA be the  | b) In what year will the population of Austin, TX and San Jose, CA be th  |
|  |   |

**8** | P a g e

| Based on recent Census Data, a logistic model for the population of |  |
|---|--|
| Dallas, TX, t years after 1900, is as follows:                      |  |

$$y = \frac{1301642}{1 + 21.602e^{-0.05054x}}$$

a) What was the population of Dallas, TX in the year 2000?

b) According to the model, what is Dallas' maximum sustainable population?

c) According to this model, when was the population 1 million.

The number of bacteria after t hours is given by  $y = 150e^{0.521t}$ a) What was the initial amount of bacteria present? b) How many bacteria are present after 4 hours? c) How many hours will it take until there are 400 bacteria?

Bacteria Growth

Chapter 3: Exponential, Logistic, and Logarithmic Functions 3.2: Exponential and Logistic Modeling

Tell whether the function is an exponential growth or decay function. Then find the constant percentage rate of growth or decay.

1. 
$$y = 10 \cdot (.7)^t$$

2. 
$$y = 5 \cdot (1.4)^t$$

3. 
$$y = 898 \cdot (1.0064)^t$$
 4.  $y = 1203 \cdot (0.9858)^t$ 

4. 
$$y = 1203 \cdot (0.9858)$$

Determine the exponential function that satisfies the given conditions

- A) Initial Value 12, increasing at the rate of 8% per year
- B) Initial Value 12, decreasing at the rate of 8% per year
- C) Initial Population 200, tripling every 5 days
- D) Initial Mass 20 grams, cutting in fourths once every 3 days

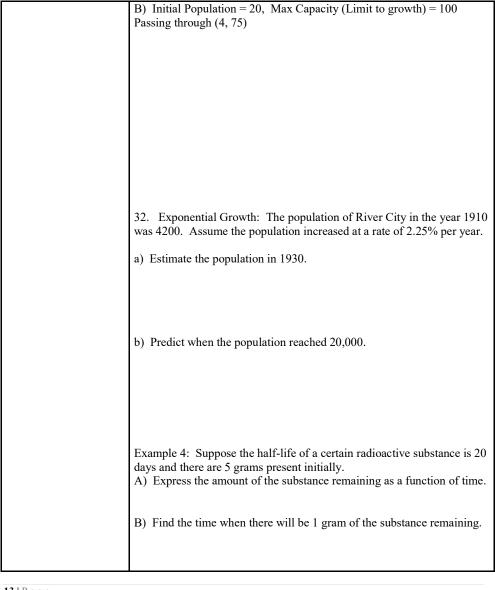
Determine a formula for the exponential function whose values are given

| X  | g(x)    |
|----|---------|
| -2 | -9.0625 |
| -1 | -7.25   |
| 0  | -5.8    |
| 1  | -4.64   |
| 2  | -3.7123 |
|    |         |

Determine a formula for the exponential function whose points are given

Find the logistic function that satisfies the given conditions

A) Initial Value 6: Max Capacity (Limit to growth) = 30 Passing through (1, 15)



Watauga High School has 1200 students. Bob, Carol, Ted and Alice start a rumor, which spreads logistically so that

 $S(t) = \frac{1200}{1 + 39e^{-0.9t}}$  models the number of students who have heard the rumor by the end of day t.

- A) How many students have heard the rumor y the end of Day 0.
- B) How long does it take for 1000 students to hear the rumor?

Use the data in the table and exponential regression to predict Dallas, TX population in 2015.

| 1950 | 434,462    |
|------|------------|
| 1960 | 679,684    |
| 1970 | 844,401    |
| 1980 | 904,599    |
| 1990 | 1,006,877  |
| 2000 | 1,1888,589 |

## PRE-CALCULUS: by Finney, Demana, Watts and Kennedy Chapter 3: Exponential, Logistic, and Logarithmic Functions

3.3: Logarithmic Functions and their graphs

| What you'l | l Learn | About |
|------------|---------|-------|
|------------|---------|-------|

Changing between Logarithmic and exponential form:

Find the inverse function for  $y = 2^x$ 

If x > 0, b > 0 and  $b \ne 1$ , then  $y = \log_b x$  if and only if  $b^y = x$ 

Properties:

If x > 0, b > 0  $b \ne 1$ , and any real number y

- any real number y •  $\log_b 1 = 0$  because  $b^0 = 1$
- $\log_b b = 1$  because  $b^1 = b$
- $\log_b b^y = y$  because  $b^y = b^y$
- $b^{\log_b x} = x$  because

$$\log_b x = \log_b y$$

 $a) \log_2 8 =$ 

b)  $\log_3 \sqrt{3} =$ 

Evaluate the logarithmic expression without using a calculator

 $c) \log_5 \frac{1}{25} =$ 

 $d \log_4 1 =$ 

e)  $\log_7 7 =$ 

15 | Page

Evaluate the logarithmic expression without using a calculator

a) 
$$\log 100 =$$

b) 
$$\log \sqrt[5]{10} =$$

c) 
$$\log \frac{1}{100} =$$

$$d$$
)  $\ln \sqrt{e} =$ 

e) 
$$\ln e^{5} =$$

$$f$$
)  $\ln \sqrt[5]{e} =$ 

Evaluate the logarithmic expression without using a calculator

a) 
$$6^{\log_6 11} =$$

$$b) 10^{\log 6} =$$

$$c) e^{\ln 4} =$$

**16** | Page

Use a calculator to evaluate the logarithmic expression if it is defined and check your result by evaluating the corresponding exponential expression

a) 
$$\log 34.5 =$$

b) 
$$\log 0.43 =$$

c) 
$$\log (-3) =$$

d) 
$$ln 23.5 =$$

e) 
$$\ln 0.48 =$$

f) 
$$ln(-5) =$$

Solve the equation

a) 
$$\log x = 3$$

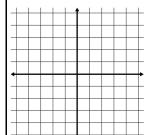
$$b) \log_2 x = 5$$

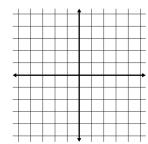
17 | Page

Describe how to transform the graph of  $y = \ln x$  into the graph of the given function. Sketch the graph by hand.

a) 
$$g(x) = \ln(x+2) + 1$$

b) 
$$h(x) = \ln(3-x)$$





- 1) Determine the vertical asymptotes
- 1) Determine the vertical asymptotes
- 2) Determine the x-intercept
- 2) Determine the x-intercept
- 3) Determine the domain and range
- 3) Determine the domain and range
- 4) Intervals of Increase or Decrease
- 4) Intervals of Increase or Decrease
- 5) Determine the end behavior
- 5) Determine the end behavior
- 6) Intervals of Concavity

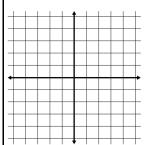
**18** | P a g e

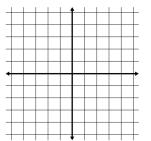
6) Intervals of Concavity

Describe how to transform the graph of  $y = \ln x$  into the graph of the given function. Sketch the graph by hand.

a) 
$$g(x) = -3 \log x$$

b) 
$$h(x) = \log(-x) - 2$$





- 1) Determine the vertical asymptotes
- 1) Determine the vertical asymptotes
- 2) Determine the x-intercept
- 2) Determine the x-intercept
- 3) Determine the domain and range
- 3) Determine the domain and range
- 4) Intervals of Increase or Decrease
- 4) Intervals of Increase or Decrease
- 5) Determine the end behavior
- 5) Determine the end behavior
- 6) Intervals of Concavity
- 6) Intervals of Concavity

**19** | Page

Chapter 3: Exponential, Logistic, and Logarithmic Functions 3.4: Properties of Logarithmic Functions

What you'll Learn About

Use your Calculator to Determine which of the following are True.

1. 
$$\log (5+2) = \log 5 + \log 2$$

1. 
$$\log (5+2) = \log 5 + \log 2$$
 2.  $\log (5\cdot 2) = \log 5 + \log 2$ 

3. 
$$\log (5 - 2) = \log 5 - \log 2$$

4. 
$$\log\left(\frac{5}{2}\right) = \log 5 - \log 2$$

$$5. \log(5 \cdot 2) = 2\log 5$$

$$6. \quad \log\left(\frac{5}{2}\right) = \frac{\log 5}{\log 2}$$

$$7. \log(5^2) = \log 5 \cdot \log 5$$

8. 
$$\log(5^2) = 2\log 5$$

9. 
$$\ln (x + 2) = \ln x + \ln 2$$

10. 
$$\log(7x) = 7\log x$$

11. 
$$\log(5x) = \log 5 + \log x$$

$$12. \quad \ln\left(\frac{x}{5}\right) = \ln x - \ln 5$$

$$13. \log\left(\frac{x}{4}\right) = \frac{\log x}{\log 4}$$

**20** | Page

$$14. \quad \log_4 x^3 = 3\log_4 x$$

Let b, R, and S are positive real numbers with  $b \ne 1$ , and c any real number

- $\bullet \log_b(RS) = \log_b R + \log_b S$
- $\bullet \log_b \left(\frac{R}{S}\right) = \log_b R \log_b S$
- $\bullet \log_b R^c = c \log_b R$

15.  $\ln(x^2) = \ln x \cdot \ln x$ 

 $16. \quad \log|4x| = \log 4 + \log|x|$ 

Prove the Product Rule for Logarithms:  $\log_b(RS) = \log_b R + \log_b S$ Let  $x = \log_b R$  and  $y = \log_b S$ 

Assuming x and y are positive, use properties of logarithms to write the expression as a **sum or difference** of logarithms or multiples of logarithms

A)  $\log(8x)$ 

B)  $\ln\left(\frac{5}{x}\right)$ 

- $C) \log_2(x^5)$
- $D) \log(8x^2y^4)$

 $E) \ln \left( \frac{\sqrt{x^2 + 5}}{\sqrt[3]{x^4}} \right)$ 

Let b, R, and S are positive real numbers with  $b \ne 1$ , and c any real number

- $\bullet \log_b(RS) = \log_b R + \log_b S$
- $\bullet \log_b \left(\frac{R}{S}\right) = \log_b R \log_b S$
- $\bullet \log_b R^c = c \log_b R$

Assuming x, y and z are positive, use properties of logarithms to write the expression as a **single** logarithm

A)  $\log x + \log 6$ 

B)  $\ln x - \ln 6$ 

 $C) \frac{1}{4} \log x$ 

- $D) 6\log x \frac{1}{2}\log y$
- $E) \quad 5\log(x^2y) + 3\log(y^2z)$

 $F) \ln x^5 - 2\ln(xy)$ 

$$\log_{64} 4096 = \frac{\log_8 4096}{\log_8 64} = \frac{4}{2} \quad A) \quad y = \log_4 7$$

$$8^{y} = 4096 \rightarrow 8^{y} = 8^{4} \quad y = 4$$
  
 $8^{y} = 64 \rightarrow 8^{y} = 8^{2} \quad y = 2$ 

Rewrite the following as an exponential function then solve for y

$$A)$$
  $y = \log_4 f$ 

$$\log_{64} 4096 = \frac{\log_4 4096}{\log_4 64} = \frac{6}{3}$$

$$4^{y} = 4096 \rightarrow 8^{y} = 4^{6} \quad y = 6$$
  
 $4^{y} = 64 \rightarrow 4^{y} = 4^{3} \quad y = 3$ 

Use the change of base formula and your calculator to evaluate the logarithm

$$A$$
)  $\log_3 16$ 

$$B) \log_{1/2} 2$$

For positive real numbers a, b and x with  $a \neq 1$  and  $b \neq 1$ 

$$\log_b x = \frac{\log_a x}{\log_a x} = \frac{\ln x}{\ln x}$$

Express using only natural logarithms

A) 
$$g(x) = \log_5 x$$

$$B) \quad g(x) = \log_2(x+y)$$

Describe the transformation of each function from the original function ln(x) or log(x)

42) 
$$f(x) = ln(x) + 2$$

44) 
$$f(x) = \ln(-x) + 2$$

46) 
$$f(x) = \ln(5 - x)$$

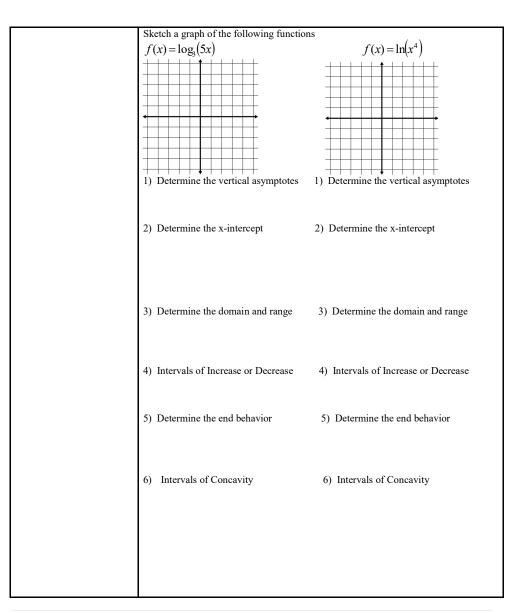
A) 
$$f(x) = \ln(x - 5)$$

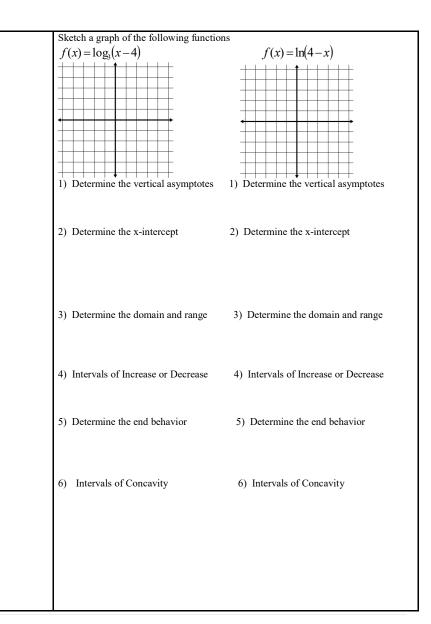
52) 
$$f(x) = -3\log(1-x) + 1$$

$$B) \ \ y = \frac{1}{3} \log x$$

C) 
$$y = log(2x)$$

$$D) \quad y = \log\left(\frac{1}{2}x\right)$$





**26** | Page

## Chapter 3: Exponential, Logistic, and Logarithmic Functions 3.5: Equation Solving and Modeling

What you'll Learn About

Find the exact solution algebraically, and check it by substituting into the original equation.

$$A) \quad \left(\frac{1}{4}\right)^x = \frac{1}{16}$$

B) 
$$20\left(\frac{1}{2}\right)^{x/3} = 5$$

C) 
$$2(3)^{x/2} = 6$$
 D)  $2(3)^{-x/2} = 54$ 

$$D) 2(3)^{-x/2} = 54$$

$$E$$
)  $log x = 5$ 

$$F) \quad \log_2(x-4) = 3$$

Solve each equation algebraically

A) 
$$2.03^x = 5$$

B) 
$$50(e)^{0.03x} = 500$$

C) 
$$2\ln(x+3)+6=1$$

C) 
$$2\ln(x+3)+6=10$$
 D)  $2-\log(x+3)=10$ 

**27** | P a g e

| Solve each equation.               |
|------------------------------------|
| $A)  \log^3 = 9$                   |
| B) $\frac{3^x - 3^{-x}}{2} = 5$    |
|                                    |
|                                    |
|                                    |
| $C) \qquad 3e^{2x} + 4e^x - 4 = 0$ |
|                                    |
|                                    |
|                                    |

| $D$ ) $\frac{700}{1+20e}$ | $\frac{0}{0.02x} = 200$ |
|---------------------------|-------------------------|
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |
|                           |                         |

| E) | $\frac{1}{2}\ln(x+2) - \ln(x) = 0$ |
|----|------------------------------------|
|    |                                    |
|    |                                    |
|    |                                    |
|    |                                    |
|    |                                    |
|    |                                    |
| F) | $\log(x+2) + \log(x+3) = 4\log 2$  |
|    |                                    |
|    |                                    |
|    |                                    |
|    |                                    |
|    |                                    |