## PRE-CALCULUS: by Finney,Demana, Watts and Kennedy

## Chapter 3: Exponential, Logistic, and Logarithmic Functions

## 3.1 : Exponential and Logistic Functions

| Exponential Function A function that can be rewritten in the form $y=a \cdot b^{x}$, where a is non-zero, $b$ is positive, and $b \neq 1$. <br> a: initial value at $\mathrm{x}=0$ <br> b: base | Which of the following are exponential functions? <br> For those that are exponential functions, state the initial value and the base. For those that are not, explain. <br> A) $f(x)=3^{x}$ <br> B) $g(x)=6 x^{-4}$ |
| :---: | :---: |
|  | $\begin{array}{ll}\text { C) } \mathrm{h}(\mathrm{x})=-2 \cdot 1.5^{x} & \text { D) } h(x)=7 \cdot-2^{x}\end{array}$ |
|  | E) $f(x)=5 \cdot 6^{\pi}$ |
|  | Compute the exact value of the function without using a calculator <br> A) $2 \cdot 4^{x}$ when $\mathrm{x}=0$ <br> B) $2 \cdot 4^{x}$ when $\mathrm{x}=-3$ |
|  | C) $-2 \cdot 4^{x}$ when $\mathrm{x}=1 / 2 \quad$ D) $3 \cdot 8^{x}$ when $\mathrm{x}=-2 / 3$ |

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2|Page

$3 \mid \mathrm{Page}$

Describe how to transform the graph of $\mathbf{f}(\mathbf{x})=\mathbf{2}^{\boldsymbol{x}}$ into the graph of $g$
a) $g(x)=2^{x-1}$
b) $g(x)=2^{-x}$
c) $g(x)=3 \cdot 2^{x}$
d) $g(x)=2^{3-x}$

Describe how to transform the graph of $\mathbf{f}(\mathbf{x})=\mathbf{e}^{\mathbf{x}}$ into the graph of g
a) $g(x)=e^{4 x}$
b) $g(x)=e^{-4 x}$
c) $g(x)=3 \cdot e^{x}+1$
d) $g(x)=e^{2-2 x}$

$\mathbf{5 | P a g e}$


6|Page

|  | Sketch a graph of the following functions $y=\frac{4}{1+2 e^{-x}}$  <br> 1) Determine the minimum and Maximum capacity (Horizontal Asy) <br> 2) Determine the $y$-intercept <br> 3) Determine the domain and range <br> 4) Intervals of Increase or Decrease <br> 5) Determine the end behavior <br> 6) Find any asymptotes <br> 7) Determine Half the max capacity <br> 8) Intervals of Concavity | $y=\frac{4}{1+2 e^{x}}$  <br> 1) Determine the minimum and Maximum capacity (Horizontal Asy) <br> 2) Determine the y-intercept <br> 3) Determine the domain and range <br> 4) Intervals of Increase or Decrease <br> 5) Determine the end behavior <br> 6) Find any asymptotes <br> 7) Determine Half the max capacity <br> 8) Intervals of Concavity |
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7|Page


|  | Based on recent Census Data, a logistic model for the population of <br> Dallas, TX, tyears after 1900, is as follows: <br> $y=\frac{1301642}{1+21.602 e^{-0.05054 \tau}}$ <br> a) What was the population of Dallas, TX in the year 2000? |
| :--- | :--- |
| b) According to the model, what is Dallas' maximum sustainable |  |
| population? |  |
| c) According to this model, when was the population 1 million. |  |


|  | Bacteria Growth <br> The number of bacteria after t hours is given by <br> $y=150 e^{0.521 t}$ |
| :--- | :--- |
| a) What was the initial amount of bacteria present? |  |
| b) How many bacteria are present after 4 hours? |  |
| c) How many hours will it take until there are 400 bacteria? |  |

10|Page

Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.2: Exponential and Logistic Modeling

$11 \mid \mathrm{Page}$

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| B) Initial Population =20, Max Capacity (Limit to growth) $=100$ |
| :--- | :--- |
| Passing through (4, 75) |$\quad$| 32. Exponential Growth: The population of River City in the year 1910 |
| :--- |
| was 4200. Assume the population increased at a rate of 2.25\% per year. |
| a) Estimate the population in 1930. |
| b) Predict when the population reached 20,000. |
| B) Find the time when there will be 1 gram of the substance remaining. |
| Example 4: Suppose the half-life of a certain radioactive substance is 20 |
| days and there are 5 grams present initially. |
| A) Express the amount of the substance remaining as a function of time. |
| Ber |

$13 \mid \mathrm{Page}$

Watauga High School has 1200 students. Bob, Carol, Ted and Alice start a rumor, which spreads logistically so that
$S(t)=\frac{1200}{1+39 e^{-0.9 t}}$ models the number of students who have heard the rumor by the end of day $t$.
A) How many students have heard the rumor y the end of Day 0 .
B) How long does it take for 1000 students to hear the rumor?

Use the data in the table and exponential regression to predict Dallas, TX population in 2015

| 1950 | 434,462 |
| :--- | :--- |
| 1960 | 679,684 |
| 1970 | 844,401 |
| 1980 | 904,599 |
| 1990 | $1,006,877$ |
| 2000 | $1,1888,589$ |

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PRE-CALCULUS: by Finney, Demana, Watts and Kennedy Chapter 3: Exponential, Logistic, and Logarithmic Functions 3.3: Logarithmic Functions and their graphs

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|  | Use a calculator to evaluate the logarithmic expression if it is defined and   <br> check your result by evaluating the corresponding exponential expression   <br> a) $\log 34.5=$ b) $\log 0.43=$ c) $\log (-3)=$ |
| :--- | :--- | :--- |
| d) $\ln 23.5=$ e) $\ln 0.48=$ f) $\ln (-5)=$ |  |
| Solve the equation  <br> a) $\log x=3$ b) $\log _{2} x=5$ |  |
|  |  |


$\mathbf{1 8 | P a g e}$


19|Page

Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.4: Properties of Logarithmic Functions

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| Let b, R, and S are positive real numbers with $\mathrm{b} \neq 1$, and c any real number <br> - $\log _{b}(R S)=\log _{b} R+\log _{b} S$ <br> - $\log _{b}\left(\frac{R}{S}\right)=\log _{b} R-\log _{b} S$ <br> - $\log _{b} R^{c}=c \log _{b} R$ | 15. $\ln \left(x^{2}\right)=\ln x \cdot \ln x$ <br> 16. $\log \|4 x\|=\log 4+\log \|x\|$ <br> Prove the Product Rule for Logarithms: $\log _{b}(R S)=\log _{b} R+\log _{b} S$ Let $x=\log _{b} R$ and $y=\log _{b} S$ <br> Assuming x and y are positive, use properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms <br> A) $\log (8 x)$ <br> B) $\ln \left(\frac{5}{x}\right)$ <br> C) $\log _{2}\left(x^{5}\right)$ <br> D) $\log \left(8 x^{2} y^{4}\right)$ <br> E) $\ln \left(\frac{\sqrt{x^{2}+5}}{\sqrt[3]{x^{4}}}\right)$ |
| :---: | :---: |

$21 \mid \mathrm{Page}$

| Let $\mathrm{b}, \mathrm{R}$, and S are positive <br> real numbers with $\mathrm{b} \neq 1$, and <br> c any real number <br> $\bullet \log _{b}(R S)=\log _{b} R+\log _{b} S$ | Assuming $\mathrm{x}, \mathrm{y}$ and z are positive, use properties of logarithms to write <br> the expression as a single logarithm |
| :--- | :--- | :--- |
| $\bullet \log _{b}\left(\frac{R}{S}\right)=\log _{b} R-\log _{b} S$ |  |
| $\bullet \log _{b} R^{c}=c \log _{b} R$ |  |$\quad$|  |  |  |
| :--- | :--- | :--- |
|  | C) $\frac{1}{4} \log x+\log 6$ | ln $x$ |

E) $5 \log \left(x^{2} y\right)+3 \log \left(y^{2} z\right)$
F) $\ln x^{5}-2 \ln (x y)$

| $\log _{64} 4096=\frac{\log _{8} 4096}{\log _{8} 64}=\frac{4}{2}$ <br> because $\begin{aligned} & 8^{y}=4096 \rightarrow 8^{y}=8^{4} \quad y=4 \\ & 8^{y}=64 \rightarrow 8^{y}=8^{2} \quad y=2 \end{aligned}$ | Rewrite the following as an exponential function then solve for y <br> A) $\mathrm{y}=\log _{4} 7$ |
| :---: | :---: |
| $\log _{64} 4096=\frac{\log _{4} 4096}{\log _{4} 64}=\frac{6}{3}$ <br> because $\begin{aligned} & 4^{y}=4096 \rightarrow 8^{y}=4^{6} \quad y=6 \\ & 4^{y}=64 \rightarrow 4^{y}=4^{3} \quad y=3 \end{aligned}$ | Use the change of base formula and your calculator to evaluate the logarithm <br> A) $\log _{3} 16$ <br> B) $\log _{1 / 2} 2$ |
| For positive real numbers $\mathrm{a}, \mathrm{b}$ and x with $\mathrm{a} \neq 1$ and $\mathrm{b} \neq 1$ $\log _{b} x=\frac{\log _{a} x}{\log _{a} x}=\frac{\ln x}{\ln x}$ | Express using only natural logarithms <br> A) $g(\mathrm{x})=\log _{5} x$ <br> B) $g(\mathrm{x})=\log _{2}(x+y)$ |

$23 \mid$ Page

|  | Describe the transformation of each function from the original function <br> $\ln (\mathrm{x})$ or $\log (\mathrm{x})$ <br> $42) \mathrm{f}(\mathrm{x})=\ln (\mathrm{x})+2$ |
| :--- | :--- |
| 46) $\mathrm{f}(\mathrm{x})=\ln (5-\mathrm{x})$ 44) $\mathrm{f}(\mathrm{x})=\ln (-\mathrm{x})+2$ |  |
| A $(\mathrm{x})=\ln (\mathrm{x}-5)$ |  |
| 52) $\mathrm{f}(\mathrm{x})=-3 \log (1-\mathrm{x})+1$ B) $\mathrm{y}=\frac{1}{3} \log x$ |  |
| C) $\mathrm{y}=\log (2 \mathrm{x})$ D) $y=\log \left(\frac{1}{2} x\right)$ |  |

24 |Page

$\mathbf{2 5 | P a g e}$


26|Page

Chapter 3: Exponential, Logistic, and Logarithmic Functions 3.5: Equation Solving and Modeling


27|Page

$\mathbf{2 8 | P a g e}$


29|Page

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